Explicit Artificial Neural Networks for Predicting Gradually Varied Flow

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Abstract

The ANN procedure was used to develop an explicit equation for predicting the water level profile in a gradually varied flow. The equation consists of a series of hyperbolic tangent functions, with the number of series being the same as the number on the node in the hidden layer. The ANN model consists of 3 layers: the input layer consists of four nodes, the hidden layer has seven nodes and one node in the output layer. The input parameters used are parameters related to distance, discharge, roughness, and depth of flow at the downstream end of the channel. The output parameter is the flow depth at various points. The model has been used to estimate the water level profile for different flow conditions. The comparison between the explicit ANN model and the numerical model results is satisfactory. The models can be extended to study more complex flows and non-prismatic channels. The model is promising as a tool in decision support.

Key Words: artificial neural networks, explicit formula, gradually varied flow

1. INTRODUCTION

The study of flow characteristics in a channel or river is an essential aspect in planning or designing processes of a regional drainage system. The engineers need to analyze the behavior of the flow in the channel due to various scenarios. Flood events, tidal effects (for a channel that is affected by tides), the impact of garbage or other materials that block the flow, water structures (gates, culverts, weirs), etc., need to be considered in the design. Two hydraulic models often used to study flow behavior in channels are physical scale and numerical models. The flow is studied with a physical scale model by observing it on a design model, which is generally made in the laboratory. Meanwhile, the study of numerical models is carried out by conducting computer simulations of several scenarios based on numerical solving of mathematical equations that explain the physical processes in a flow.

Numerical models are currently prevalent because of several advantages: relatively cheap, fast, and flexible. Several numerical models are available, including HEC-RAS, MIKE 11, DUFFLOW, etc. Numerical models require input consisting of geometry (cross section and bed slope), input discharge such as flood discharge and runoff, boundary conditions at the ends of the channel, channel roughness, water structures, etc. By entering the input parameters into the numerical model, a modeler can study the flow behavior through simulation for various desired scenarios. Users of numerical models must have good skills in modelling and understanding of hydraulics. What about engineers who have no skills in numerical modeling going to study flow behavior in channels? This obstacle can be overcome if an explicit model that connects the input and output parameters relating to input flow and channel geometry is available.

This study proposes an explicit equation that represents the channel’s flow pattern. The equations are built using an artificial neural network approach. This study aims to provide a method for developing an explicit equation for the flow pattern in a channel based on an Artificial Neural Network (ANN) procedure. The equations are built using a numerical model based on many data (big data). The study is limited to the problem of steady flow in prismatic channels. Further development of explicit equations can be carried out for unsteady and non-prismatic flows using the same principle. The following sections will discuss the numerical model of gradual changing flow, a brief explanation of ANN, explicit equation development methodology, model simulation, and discussion and conclusions.

2. METHODOLOGY

Numerical Model

Consider a channel diagram and flow conditions in Figure 1. The subcritical steady gradually varied flow equation can be expressed by equation (1) below (Subramanya, 2009).

\[
\frac{dh}{dx} = \frac{s_o - s_f}{1 - F_o^2} = F(x, h)
\]

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with
\[ S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} \]
\[ F_r = \frac{Q^2 R}{g A^3} \]
\[ T = B + 2mh \]
\[ A = (B + mh)h, R = \frac{A}{P} \]
\[ P = B + 2h\sqrt{1 + m^2} \]

where \( B \) = bottom width, \( h \) = flow depth, \( m \) = cross-sectional side slope, \( S_o \) = bed slope, \( S_f \) = flow energy slope, \( A \) = wet cross-sectional area, \( P \) = wet circumference, \( R \) = hydraulic radius, \( F_r \) = Froude number, \( x \) = distance. Equation (1) can be solved in various ways. The often-used methods are the standard step method and finite difference technique such as Euler, modified Euler, and Runge-Kutta methods (Chapra and Canale, 2010; Subramanya, 2009). We use the modified Euler technique for the discretization of equation (1). First, the length of the channel is divided into several segments with the width of the segment \( \Delta x \). Let \( h_i \) is known flow depth at point \( x_i \), then the flow depth at point \( x_{i+1}, h_{i+1} \) can be determined using equation (2.a)–(2.d) (Subramanya, 2009).

\[ h_{i+1} = h_i + 0.5(k_o + k_2)\Delta x \quad (2.a) \]
\[ k_o = F(x_i, h_i) \quad (2.b) \]
\[ h_i^p = h_i + k_o\Delta x \quad (2.c) \]
\[ k_1 = F(x_{i+1}, h_i^p) \quad (2.d) \]

where \( h_i \) and \( h_{i+1} \) are the flow depths at points \( x_i \) and \( x_{i+1} \), respectively. The calculation starts from the segment containing the boundary conditions, it can start from the end of the upstream or downstream boundary conditions.

**Explicit ANN Formulation**

Artificial Neural Networks is a black-box model that connects input and output parameters. According to Hu et al. (2018), the Artificial Neural Network (ANN) function imitates the neural working process of the human brain, which is a form of Artificial Intelligence (AI). ANN can be trained with datasets into models that can predict and prepare their intrinsic relationships. The ANN model is an efficient tool to display nonlinear and complex relationships between inputs and outputs. The use of ANN model has been widely used in water engineering. For example: in suspended sediment estimations (Cigizoglu, 2006; Kisi, 2008), in the prediction of bed sediment load (Sahraei, et al., 2017; Riahi-Madvar and Seifi, 2018), in river flow study (Adnan et al., 2019; Aqil et al., 2007; Cheng et al., 2020; Dibiki and Solomatine, 2001; Fu et al. 2020; Huo et al. 2012; Imrie et al. 2000; Liu et al., 2020; Ni et al. 2010; Noori and Kalin, 2016; Shamshirband et al., 2020; Xang and Demir 2020), in rainfall-runoff modeling (Xiang et al., 2020), flood predicting (Mosavi and Ozturk, 2018), hydraulic jump (Omid et al., 2005), etc. The ANN model is implicit, and for those who are not familiar with ANN, it won’t be easy to understand and apply it in the practices. Therefore, in this study, an explicit form of the ANN model will be developed so that it is easy to use for those who are not familiar with ANN.

The often-used ANN model is a feed-forward neural network with three layers: input, hidden, and output. Each layer has several nodes (neurons) that are connected to other nodes in the next layer. The model has a feed-forward phase that propagates the input signal forward to each node in the front layer until it reaches the output layer, and the error propagates back (error backward propagation) and modifies the connection relationship between nodes (weight). Error is defined as the difference between the calculated value and the observed value of the target variable. The input parameters used in the flow equation of ANN modeling in this study are \( B, x, Q, S_o, n, h_o \), while the output parameter is the flow depth \( h \). We consider channels with specific geometries with fixed \( S_o \) and \( B \) values or set fixed cross-sectional shapes. Thus, the input parameter consists of \( Q, x, n, \) and \( h_o \), while the output parameter is the flow depth \( h \). The ANN architectural model under review is illustrated in Figure 2.

**Figure 1.** Schematic for gradually varied flow together with flow parameters and channel geometry.

**Figure 2.** Schematic diagram of ANN configuration, consisting of 4 inputs, 7 nodes at hidden layer, and a single output node.

The ANN model with configuration in Figure 2 can be written in mathematics form, defined by Equation (3).
\[
    h_A = G(\sum_{k=1}^{M} \mu_k H(\sum_{i=1}^{N} a_i \bar{X}_i + \beta_i \bar{X}_2 + \theta_i \bar{X}_3 + \omega_i \bar{X}_4 + b_i) + \mu_{M+1})
\]

(3)

with \( \bar{X}_i \) are input values in the normalized form at node \( i \), \( h_A \) is output, \( g_i \) is activation function (nonlinear) for the hidden layer, \( g_2 \) is activation function (linear) for output layer, \( N \) and \( M \) represent the number of neurons in the input and hidden layers, \( a_i, \beta_i, \theta_i \) and \( \omega_i \) and \( b_i \) are the weights and biases of the \( i^{th} \) neuron in the input layer to the hidden layer, and \( \mu_k \) are the weights and biases from the hidden layer to the output layer.

Before entering into the network, the input parameters need to be normalized using the following equation.

\[
    \bar{X}_i = \frac{X_i - X_{i_{\text{min}}}}{X_{i_{\text{max}}} - X_{i_{\text{min}}}}
\]

(4)

where \( X_{i_{\text{min}}} \) and \( X_{i_{\text{max}}} \) are the minimum and maximum values of the input parameters \( X_i \). Due to space limitations, the ANN model is not described in detail in this paper. Interested readers can take a look at some of the literature (Haykin, S., 1999; Rojas, R., 1996)). The weight and bias values \( a_i, \beta_i, \theta_i \) and \( \omega_i, b_i \) and \( \mu_k \) are obtained through the training process. In this study, the functions \( g_i \) and \( g_2 \) used are hyperbolic tan transformation and linear functions. Setting equation (3) will further produce an equation of the relationship between the water depth and the input parameter \( X_i \), which is defined by equation (5).

\[
    h_A = \sum_{i=1}^{M} \mu_i \tanh(a_i \bar{X}_1 + \beta_i \bar{X}_2 + \theta_i \bar{X}_3 + \omega_i \bar{X}_4 + b_i) + \mu_{M+1}
\]

(5)

3. DATA GENERATING AND SIMULATION

Creating a database as an input parameter is carried out by simulating the numerical model of equation (2) for various scenarios. This study reviews two-channel geometries with the same bottom width \( B = 30 \) and the bed slope \( S_0 = 0.005 \) and \( S_0 = 0.001 \). For each channel, seven variations of the quantity of discharge \( Q \) were chosen, namely 100, 150, 200, 250, 300, 350 and 400 m/s, six roughness quantities \( n \), starting from \( n = 0.025, 0.03, 0.035, 0.04 \) and 0.045, 30 values of water depth at downstream \( h_0 \), starting from \( h_0 = 3.1 \) to \( h_0 = 6 \) m with an interval of 0.1 m increments, and 152 variations of \( x \) values, starting from \( x = 1, 3, 5, 10, 20, 30, 40, 50 \) and so on with relationship \( x_i+1 = x_i + 20 \) m until it reaches the length of the farthest point \( x = 750 \) m. Each channel generates 52,950 big data. Each data contains parameters \((B, S_0, x, Q, n, h_0, h)\). The selection of the maximum value of \( x = 750 \) m is necessary in order to avoid too much data for \( x \) values which can result in overfitting in the training process. Simulations for \( x \) values greater than 750 m can be performed using the method. The final value of \( h \) at the previous simulation at \( x = 750 \) m is used as \( h_0 \). Moreover, the data for validation and testing is 5,040 data built with four variations of discharge, from \( Q = 125, 175, 230, \) and 320 m/s, three variations of roughness \( n \) from \( n = 0.027, 0.033, \) and 0.042, and 10 variations of \( h_0 \) from 3 to 4.0 m. Variation of \( x \) is as in the creation of data for training.

The ANN model requires suitable input parameters for better results. If the input parameters used are primitive, \( x, Q, h_0 \), and \( n \), the results are not satisfactory. After some numerical tests, the best input parameters are \( X_1=x/h_0, X_2=x/Q, X_3=x/h, \) and \( X_4=h_0 \), and the output parameter is the flow depth at point \( x_i, h_i \). The Neural Network Toolbox of MATLAB software was used in our analysis, with the Levenberg-Marquardt (trainlm) being adopted. The author has also developed a program based on the above ANN algorithm using an MS Excel spreadsheet to create the proposed explicit formulas defined in equation (5) using weight values solved by the MATLAB software. The optimization results produce the coefficient of equation (5). Tables 1 and 2 contain the coefficients Eq. (5) for each channel with \( S_0 = 0.0025 \) and \( S_0 = 0.001 \), respectively. Figures 3 and 4 show scatter plots between simulation results using numerical models and ANN for training data and validation & testing data for channels with \( S_0 = 0.0005 \). Figures 5 and 6 show the plot of the corresponding values for the second channel with \( S_0 = 0.001 \). The training, validation, and testing results provide a good correlation for both channels, with a correlation value of \( R^2 \) above 0.99.

Table 1. Coefficient of Eq. (5) for bed slope \( S_0 = 0.0005 \) and \( B = 30 \) m.

<table>
<thead>
<tr>
<th>( i/k )</th>
<th>( a_i )</th>
<th>( \beta_i )</th>
<th>( \theta_i )</th>
<th>( \omega_i )</th>
<th>( b_i )</th>
<th>( \mu_i )</th>
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Table 2. Coefficient of Eq. (5) for bed slope \( S_0 = 0.001 \) and \( B = 30 \) m.

<table>
<thead>
<tr>
<th>( i/k )</th>
<th>( a_i )</th>
<th>( \beta_i )</th>
<th>( \theta_i )</th>
<th>( \omega_i )</th>
<th>( b_i )</th>
<th>( \mu_i )</th>
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<tr>
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</table>
These results indicate that the ANN model of equation (5) can be used to predict the flow profile in the channel. The following shows the use of the ANN model equation (5) to predict the flow profile in the channel. The first case is the first channel with discharge $Q = 150 \text{ m}^3/\text{s}$, $n = 0.025$ and $h_0 = 4.0 \text{ m}$. Simulations were carried out along 2000 m. Simulations to determine the flow profile from location $x = 700 \text{ m}$ to $x = 2000 \text{ m}$ were carried out using the technique described previously. The predicted flow depth at $x = 700$ is used as the $h_0$ value for the next simulation. The value of $x = 0$ starts from the endpoint of the first simulation, namely $x = 700$. The simulations from $x = 1400$ to $x = 2000$ were carried out using the same procedure. The simulation results are presented in Figure 7 and 8. It can be seen from Figures 7 and 8 the prediction results of the ANN model Eq. (5) are very satisfactory, where the ANN prediction results profile and numerical model coincide.
The second example is for the same channel with discharge $Q = 350$ m$^3$/s, $n = 0.025$ and $h_o = 3.5$ m. The simulation results are shown in Figure 9. As before, the simulation results are very satisfactory. The next example is the second channel, $S_o = 0.001$ with discharge $Q = 300$ m$^3$/s, $n = 0.033$ and $h_o = 4.5$ m. The simulation results are shown in Figures 10 and 11. Like the previous example, the simulation gives accurate results. The last example is for discharge $Q = 400$ m$^3$/s, $n = 0.033$ and $h_o = 3.2$ m. The simulation results are very accurate, as shown in Figure 12. The simulation results of these cases show that the explicit form of the ANN model in equation (5) can be used to estimate the water level profile in the channel. The use of Eq. (5) is straightforward, and calculations can be done in a spreadsheet without coding or programming.
The ANN model Eq. (5) can be developed for the study of more complex flow behavior, such as unsteady flow in non-prismatic channels. The model development procedure is the same. First of all, a numerical model is used to simulate the flow conditions in the river based on the available flow and geometries data. Numerical models need to be calibrated. After the numerical model is calibrated, then several model simulations are carried out for the various scenarios considered, such as flooding, gate operation failure, tidal effect, etc. The simulation produces an extensive database, which is used to build the ANN model. The ANN model obtained will help to predict the flow conditions that will occur due to changes in input to the river. Thus, the ANN model will be advantageous as a decision support tool because it can quickly provide real-time solutions.

4. CONCLUSION

An equation for predicting the water level profile in a gradually varied flow has been developed using the ANN procedure. The equation obtained is an explicit form of the ANN model. The ANN model consists of 3 layers: the input layer consists of four nodes, the hidden layer has seven nodes and one node in the output layer. The ANN model was built using a database obtained from numerical model simulation results for various flow parameters, such as discharge, roughness, flow height at the channel’s downstream end, and the distance and flow depth along the channel. The input parameters used are parameters related to distance \( x \), discharge, roughness, and depth of flow at the downstream end of the channel. The output parameter is the flow depth at various points. The model has been used to estimate the water level profile in the channel for different flow conditions. The results of the comparison between the explicit ANN model and the numerical model results are very satisfactory, where the water level profiles of the two models are almost identical. Models can be developed for more complex flows and on non-prismatic channels. The explicit ANN model is promising to be used as a decision support tool.

REFERENCE


